Decidable Reasoning in a **First-Order** Logic of Limited **Conditional Belief**

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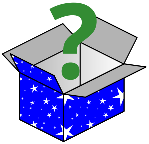
[3] Supported by a EurAI travel grant

Reasoning in conditional KBs

All we believe:



- The box is empty
- If it's *not* empty, it contains only gifts
- —— 〃 —— it contains nothing broken

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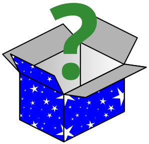

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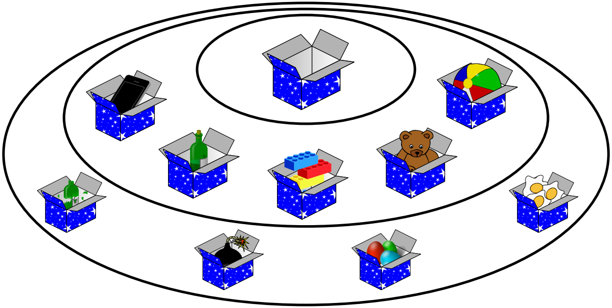

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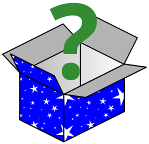

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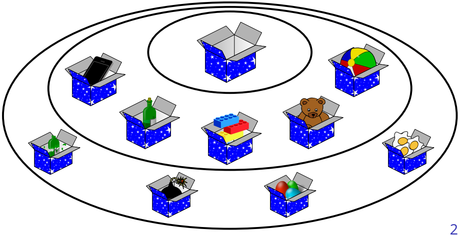
Reasoner is **omniscient** $\rightarrow$ **undecidable** (f.o.), **intractable** (prop.)



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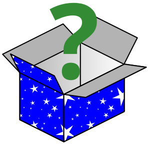

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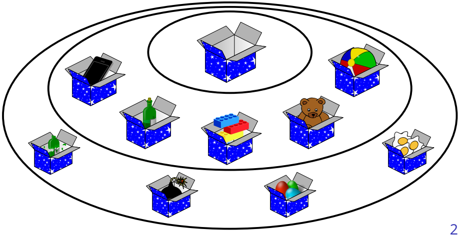
Limited reasoning to keep entailments decidable / tractable



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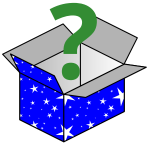

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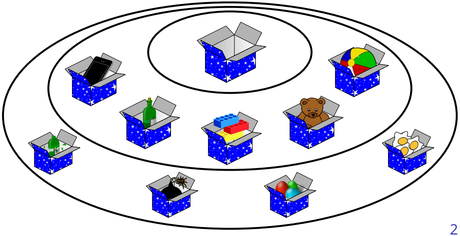
sets of clauses + case splitting
unit propagation + subsumption



Logic of conditional belief

First-order predicate logic with two modal operators:

- $\mathbf{B}(\alpha \Rightarrow \beta)$ $\hat{=}$ we believe that if α, then β
- $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ $\hat{=}$ *all* we believe is $\{\alpha_i \Rightarrow \beta_i\}$ a.k.a. only-believing

Belief entailment

Does $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ entail $\mathbf{B}(\alpha \Rightarrow \beta)$?

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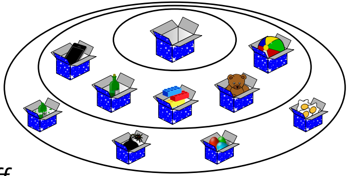
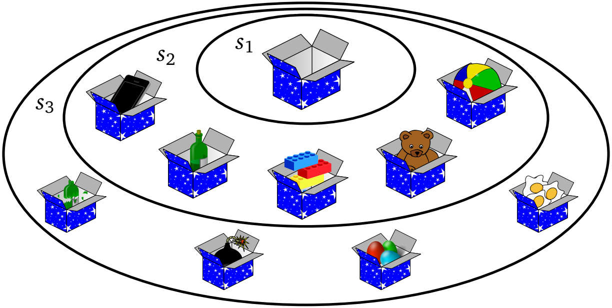


Semantics wrt system of spheres $\vec{s}$:

- $\vec{s}$ satisfies $\mathbf{B}(\alpha \Rightarrow \beta)$ *iff* the most-plausible α-worlds satisfy β
- $\vec{s}$ satisfies $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ *iff*
  - (i) $\vec{s}$ satisfies all $\mathbf{B}(\alpha_i \Rightarrow \beta_i)$
  - (ii) $\vec{s}$ is maximal subject to (i)

Example of limited conditional belief

$$\mathbf{O}\{\text{True} \Rightarrow \forall x\, \neg \text{InBox}(x),$$
$$\exists y\, \text{InBox}(y) \Rightarrow \forall x\, (\text{InBox}(x) \supset \text{Gift}(x)),$$
$$\exists y\, \text{InBox}(y) \Rightarrow \forall x\, (\text{InBox}(x) \supset \neg \text{Broken}(x))\}$$



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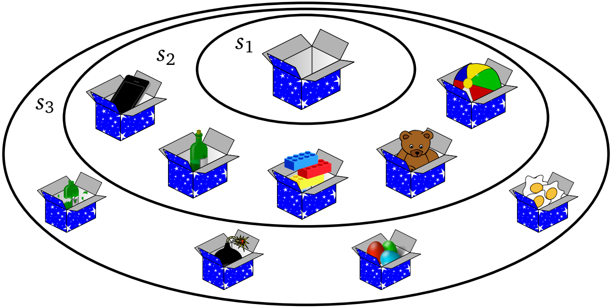
set of individuals

$$s_1 = \{\neg \mathrm{InBox}(n) \mid n \in N\}$$
$$s_2 = \{\neg \mathrm{InBox}(n) \vee \mathrm{Gift}(n),\ \neg \mathrm{InBox}(n) \vee \neg \mathrm{Broken}(n) \mid n \in N\}$$
$$s_3 = \{\}$$



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Does $\vec{s}$ satisfy $\mathbf{B}(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \land \neg\mathrm{Broken}(n))$?

- s_1 is not consistent with $\mathrm{InBox}(n)$
  - s_1 contains $\neg\mathrm{InBox}(n)$

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Does $\vec{s}$ satisfy $\mathbf{B}(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \wedge \neg\mathrm{Broken}(n))$?

- s_2 is consistent with $\mathrm{InBox}(n)$
  - $s_2 \cup \{\mathrm{InBox}(n)\}$ contains $\mathrm{InBox}(n)$
  - $s_2 \cup \{\mathrm{InBox}(n)\}$ mentions no lit. pos+neg after unit prop., subsumption

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belief level ≙ added literals

Does $\vec{s}$ satisfy $\mathbf{B}_1(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \wedge \neg \mathrm{Broken}(n))$? ✓

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belief level $\hat{=}$ added literals

Does $\vec{s}$ satisfy $\mathbf{B}_0(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \land \neg \mathrm{Broken}(n))$? ✗

Does $\vec{s}$ satisfy $\mathbf{B}_1(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \land \neg \mathrm{Broken}(n))$? ✓

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Logic of limited conditional belief

Language as before plus reasoning effort $k \in \{0, 1, 2, ...\}$:

- $\mathbf{B}_k(\alpha \Rightarrow \beta)$ $\hat{=}$ belief at level k
- $\mathbf{O}_k\{\alpha_1 \Rightarrow \beta_1, ..., \alpha_m \Rightarrow \beta_m\}$ $\hat{=}$ only-belief at level k
  prenex-NNF of $\alpha_i \supset \beta_i$ is $\forall$-clause

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Does $\mathbf{O}_k\{\alpha_1 \Rightarrow \beta_1, ..., \alpha_m \Rightarrow \beta_m\}$ entail $\mathbf{B}_{k'}(\alpha \Rightarrow \beta)$?

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Semantics for $\mathbf{B}_k$ and $\mathbf{O}_k$ as before except:

- Sets of worlds $\rightarrow$ sets of ground clauses closed under subsumption and unit propagation
- Sound but incomplete consistency test
- —— " —— satisfaction test

Limited belief entailment: sound and decidable

Soundness

If $\mathbf{O}_k\{\alpha_1 \Rightarrow \beta_1, \dots, \alpha_m \Rightarrow \beta_m\}$ entails $\mathbf{B}_{k'}(\alpha \Rightarrow \beta)$, then $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \dots, \alpha_m \Rightarrow \beta_m\}$ entails $\mathbf{B}(\alpha \Rightarrow \beta)$

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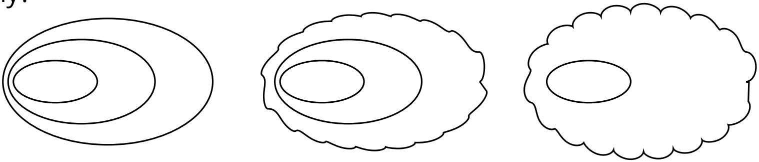
$\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ entails $\mathbf{B}(\alpha \Rightarrow \beta)$

Why?



Unlimited system of spheres — Limited approximation (good) — Limited approximation (bad)

Limited belief entailment: sound and decidable

Complexity

Whether $\mathbf{O}_k\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ entails $\mathbf{B}_{k'}(\alpha \Rightarrow \beta)$ is

- First-order case: decidable
- Propositional case: tractable for fixed effort k, k'

Summary: limited conditional belief

- Limited reasoning . . .
  - limits inferences by reasoning effort bound
  - avoids syntactic restrictions (in the query)
- Limited belief entailment is . . .
  - sound (w.r.t. unlimited logic)
  - decidable (first-order)
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What's next?

- Implementation
- Functions
- Limited revision
- Actions
- Introspection