A Semantic Account of Iterated Belief Revision in the Situation Calculus

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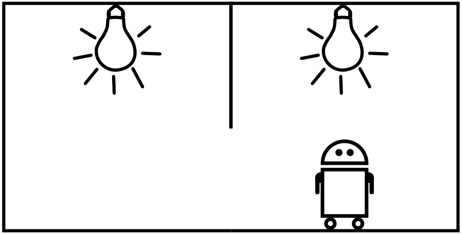
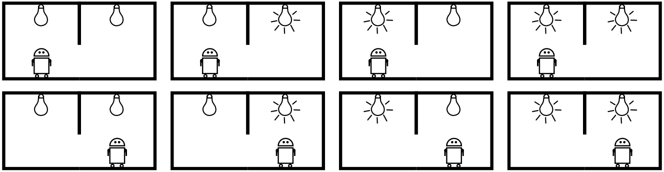
ECAI-2014, Prague

*Thanks for the money, ECCAI.

Action Theory meets Belief Revision



Robot's possible worlds:



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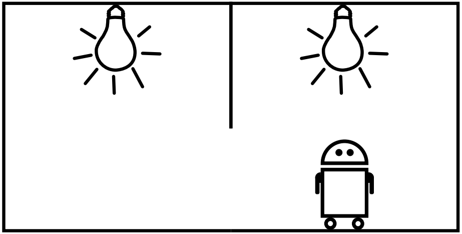
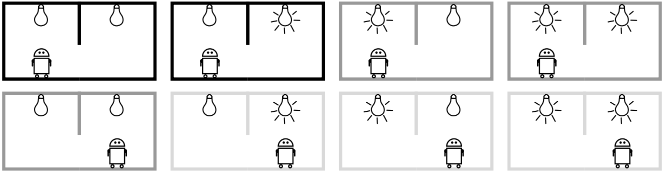
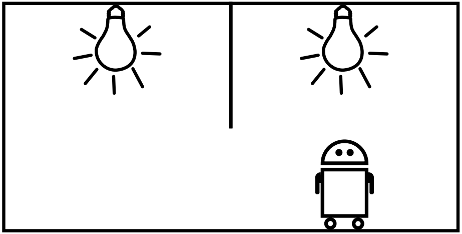
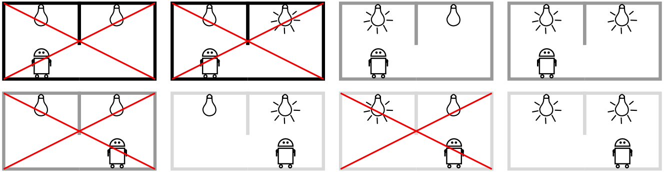


Differently plausible:





Sensing the light:



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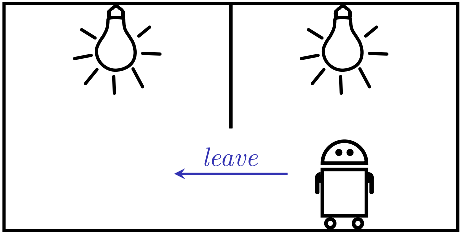
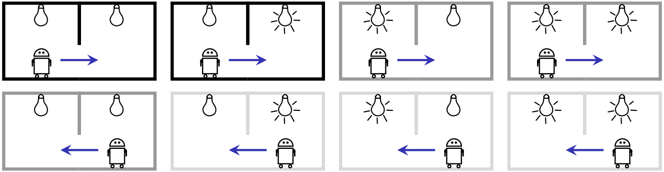


Project actions onto beliefs:



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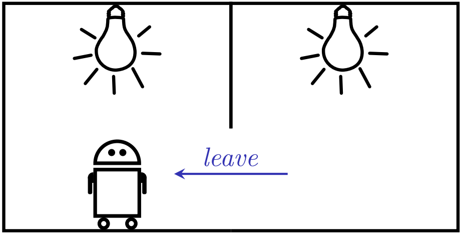

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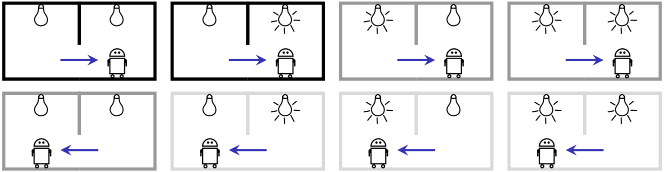


Why?

Differences to / advantages over

Shapiro, Pagnucco, Lespérance, Levesque
Iterated Belief Change in the Situation Calculus
Artificial Intelligence, **175**(1), 2011

- Semantic instead of axiomatic
  - No possible worlds / plausibilities / situations in the language
  - Simplifies reasoning
- Only-believing
  - No negated belief conditionals
  - Much smaller KBs

Language

- First-order modal language
- $[t]\alpha$ — α holds after action t
- $\Box\alpha$ — α holds after any action sequence
- $\mathbf{K}\alpha$ — we know α
- $\mathbf{B}\alpha$ — we believe α
- $\mathbf{B}(\phi \Rightarrow \psi)$ — we believe that, if ϕ held, then ψ would hold
- $\mathbf{O}(\alpha, \{\phi_1 \Rightarrow \psi_1, \ldots, \phi_m \Rightarrow \psi_m\})$
  we know α and believe $\phi_i \Rightarrow \psi_i$, but nothing else

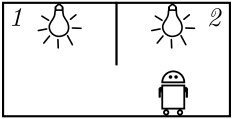
Models

$$f, w, z \models \alpha$$

- z = sequence of executed actions
- w = real world — atomic sentences $\times$ action sequences $\rightarrow \{0, 1\}$
- f = epistemic state — plausibilities $\mathbb{N} \rightarrow$ sets of possible worlds

We omit f or w when irrelevant and z when empty

Real World



$$w \models \neg R_1 \wedge L_1 \wedge L_2 \wedge$$

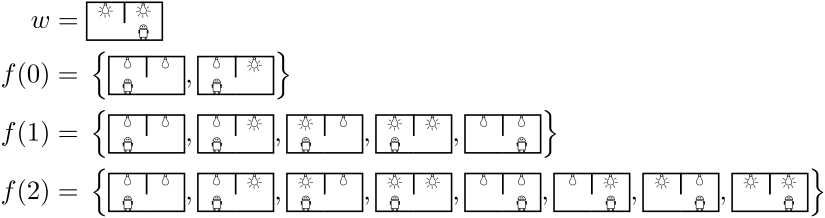
$$\big(\underbrace{\Box[a]R_1 \equiv (a = leave \wedge \neg R_1) \vee (a \neq leave \wedge R_1)}_{\text{successor state axiom}}\big) \wedge$$

$$\big(\underbrace{\Box SF(senseL) \equiv (R_1 \wedge L_1) \vee (\neg R_1 \wedge L_2)}_{\text{sensed fluent axiom}}\big)$$

- $w \models [leave]R_1$
- $w \models [leave][senseL]R_1$

Believing

$f, w, z \models \mathbf{B}\alpha$	iff	α holds at first non-empty plausibility level
$f, w, z \models \mathbf{B}(\phi \Rightarrow \psi)$	iff	$\phi \supset \psi$ holds at first plausibility level consistent with ϕ



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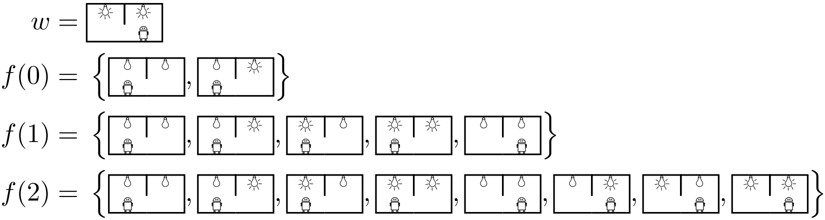

- $f, w \models \mathbf{B}\neg L_1$

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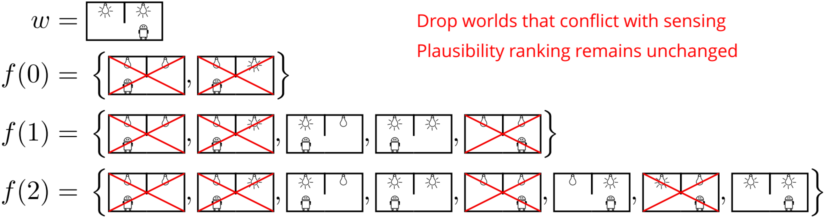

Drop worlds that conflict with sensing

Plausibility ranking remains unchanged

- $f, w \models \mathbf{B}\neg L_1$
- $f, w \models [senseL]\mathbf{B}(R_1 \wedge L_1)$

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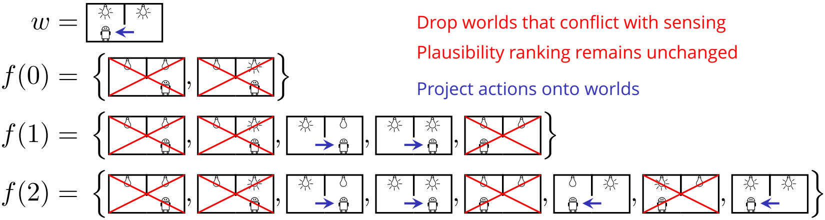

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Project actions onto worlds

- $f, w \models \mathbf{B}\neg L_1$
- $f, w \models [senseL]\mathbf{B}(R_1 \wedge L_1)$
- $f, w \models [senseL][leave]\mathbf{B}(\neg R_1 \wedge L_1)$

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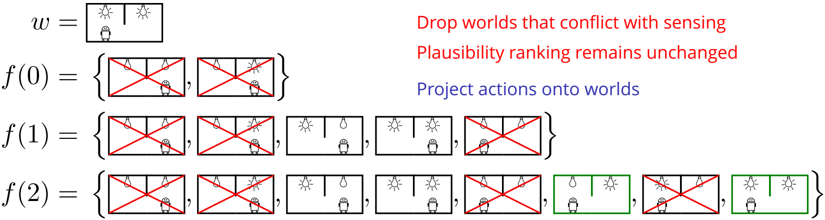

- $f, w \models \mathbf{B}\neg L_1$
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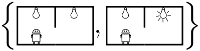
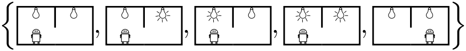
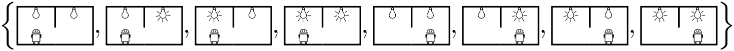
- $f, w \models [senseL][leave]\mathbf{B}(R_1 \Rightarrow L_2)$

Only-Believing

How to axiomatize f?

$f(0) =$ 

$f(1) =$ 

$f(2) =$ 

Answer:

$$f \models \mathbf{O}(\delta, \{\, \mathit{True} \Rightarrow R_1 \land \neg L_1,\ \ L_1 \Rightarrow R_1,\ \ \neg R_1 \Rightarrow \neg L_2 \})$$

where $\delta = \big(\Box[a]R_1 \equiv \ldots\big) \land \big(\Box SF(\mathit{senseL}) \equiv (R_1 \land L_1) \lor (\neg R_1 \land L_2)\big) \land \ldots$

Only-Believing

$f, w, z \models \mathbf{O}(\alpha, \{\phi_1 \Rightarrow \psi_1, \ldots, \phi_m \Rightarrow \psi_m\})$ iff

(1) α holds at all plausibility levels

(2) $\phi_i \supset \psi_i$ holds at all plausibility levels up to the first consistent with ϕ_i

(3) all plausibility levels are maximal sets of worlds

Unique-Model Property

Let α, ϕ_i, ψ_i be objective. Then:

$$f \models \mathbf{O}(\alpha, \{\phi_1 \Rightarrow \psi_1, \ldots, \phi_m \Rightarrow \psi_m\}) \text{ exists and is unique.}$$

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Let $p_1 := 0, \ldots, p_m := 0$.

For p in $0, \ldots, m$:

Let $f(p) := \{w \mid w \models \alpha \land \bigwedge_{i:p_i \geq p}(\phi_i \supset \psi_i)\}$.

For all i, if there is no $w \in f(p)$ with $w \models \phi_i$, let $p_i := p + 1$.

For all $p > m$, let $f(p) := f(m)$.

Only-Believing

$$\delta = \big(\Box[a]R_1 \equiv \ldots\big) \wedge \big(\Box SF(senseL) \equiv (R_1 \wedge L_1) \vee (\neg R_1 \wedge L_2)\big) \wedge \ldots$$

$$\Gamma = \big\{ True \Rightarrow R_1 \wedge \neg L_1,\; L_1 \Rightarrow R_1,\; \neg R_1 \Rightarrow \neg L_2 \big\}$$

$$\boxed{f \models \mathbf{O}(\delta, \Gamma)}$$

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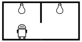
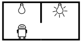
$$\boxed{f \models \mathbf{O}(\delta, \Gamma)} \qquad \text{iff}$$

$$f(0) = \{w \mid w \models \delta \wedge (True \supset R_1 \wedge \neg L_1) \wedge (L_1 \supset R_1) \wedge (\neg R_1 \supset \neg L_2)\}$$

$$= \{w \mid w \models \delta \wedge R_1 \wedge \neg L_1\}$$

$= \{$ ,  $\}$

Only-Believing

$$\delta = (\Box[a]R_1 \equiv \dots) \land (\Box SF(senseL) \equiv (R_1 \land L_1) \lor (\neg R_1 \land L_2)) \land \dots$$

$$\Gamma = \{\mathit{True} \Rightarrow R_1 \land \neg L_1,\ L_1 \Rightarrow R_1,\ \neg R_1 \Rightarrow \neg L_2\}$$

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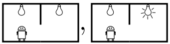
$$= \{w \mid w \models \delta \land R_1 \land \neg L_1\}$$

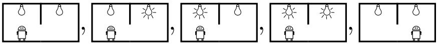
$$= \{\ \dots\ \}$$



$$f(1) = \{w \mid w \models \delta \land (L_1 \supset R_1) \land (\neg R_1 \supset \neg L_2)\}$$

$$= \{w \mid w \models \delta \land (R_1 \lor (\neg L_1 \land \neg L_2))\}$$

$$= \{\ \dots\ \}$$



Only-Believing

$$\delta = (\Box[a]R_1 \equiv \dots) \wedge (\Box SF(senseL) \equiv (R_1 \wedge L_1) \vee (\neg R_1 \wedge L_2)) \wedge \dots$$

$$\Gamma = \{True \Rightarrow R_1 \wedge \neg L_1,\ L_1 \Rightarrow R_1,\ \neg R_1 \Rightarrow \neg L_2\}$$

$$f \models \mathbf{O}(\delta, \Gamma)$$

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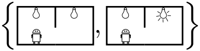
$=$ 

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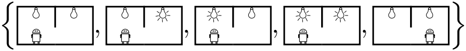
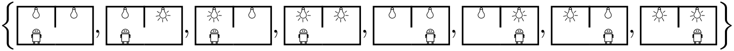
$=$ 

$$f(2) = \{w \mid w \models \delta\}$$

$=$ 

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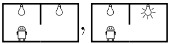
$$f \models \mathbf{O}(\delta, \Gamma)$$

iff

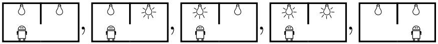
$$f(0) = \{w \mid w \models \delta \land (True \supset R_1 \land \neg L_1) \land (L_1 \supset R_1) \land (\neg R_1 \supset \neg L_2)\}$$

$$= \{w \mid w \models \delta \land R_1 \land \neg L_1\}$$

$= \{$  $\}$

$$f(1) = \{w \mid w \models \delta \land (L_1 \supset R_1) \land (\neg R_1 \supset \neg L_2)\}$$

$$= \{w \mid w \models \delta \land (R_1 \lor (\neg L_1 \land \neg L_2))\}$$

$= \{$  $\}$

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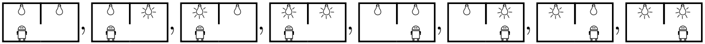
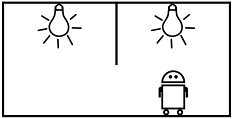
$= \{$  $\}$

$f(p) = f(2)$ for $p > 2$

Believing vs Only-Believing



Let $\mathbf{B}\Gamma \doteq \bigwedge_{\phi \Rightarrow \psi \in \Gamma} \mathbf{B}(\phi \Rightarrow \psi)$

- $\models \delta \wedge \gamma \wedge \mathbf{O}(\delta, \Gamma) \supset [senseL]\mathbf{B}(R_1 \wedge L_1)$ but
  $\not\models \delta \wedge \gamma \wedge \mathbf{K}\delta \wedge \mathbf{B}\Gamma \supset [senseL]\mathbf{B}(R_1 \wedge L_1)$

  Reason: $\mathbf{B}\Gamma$ allows "holes" in plausibility ranking

- $\models \mathbf{O}(\alpha, \Gamma) \supset \mathbf{K}\alpha \wedge \mathbf{B}\Gamma$ but
  $\not\models \mathbf{K}\alpha \wedge \mathbf{B}\Gamma \supset \mathbf{O}(\alpha, \Gamma)$

Conclusion

- Possible worlds / situations / plausibilities purely semantic
- $\mathbf{O}(\alpha, \Gamma)$ has a unique model and
  only finitely many relevant levels
- No need for *negated* conditionals $\neg\mathbf{B}(\phi \Rightarrow \psi)$ in KB

$$\models \delta \wedge \gamma \wedge \mathbf{O}(\delta, \Gamma) \supset [z]\mathbf{B}\alpha$$

Next:

- Projection problem
  - Regression
  - Progression
- Limited reasoning [Lakemeyer and Levesque, KR-2014]
  - Decidable fragment
  - Implementation
- Changing plausibility ranking [Delgrande and Levesque, KR-2012]

Appendix

Postulates

Inherit results from Shapiro et al. [2011]

- AGM postulates 1-8 hold
- KM postulates 1,2,4,5,8 hold
- DP postulates 1,3,4 hold