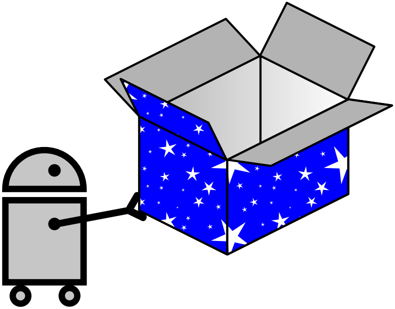
Conditional Beliefs in Action

Christoph Schwering

What's in the box?



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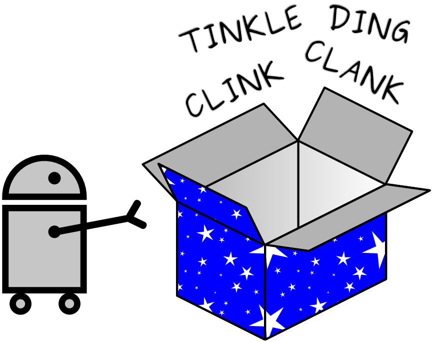

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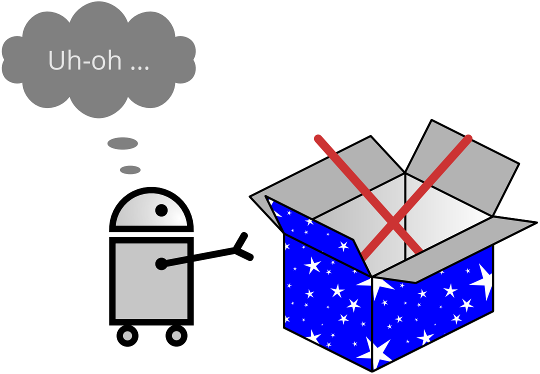

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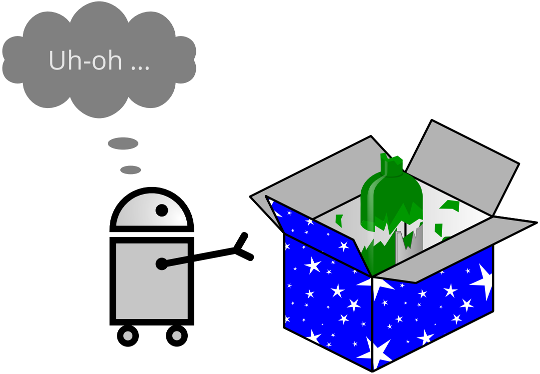

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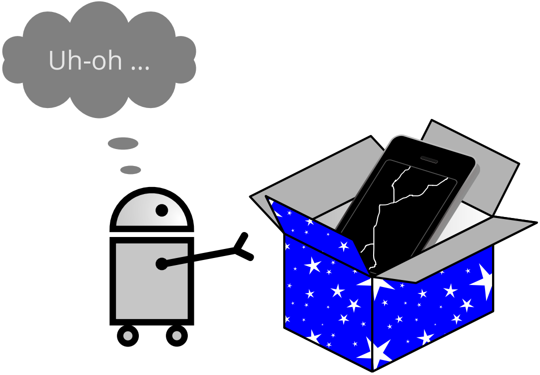

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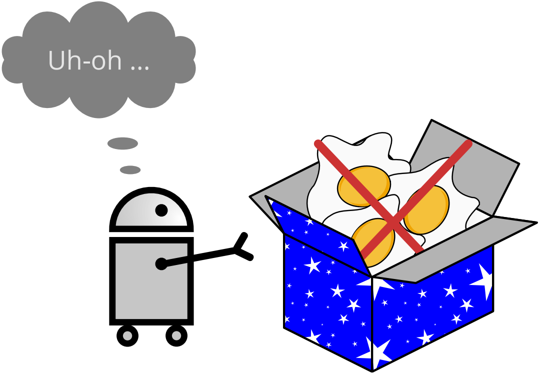

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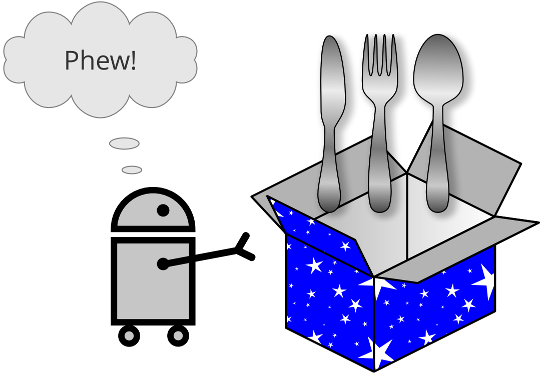

What's in the box? — Thesis objective

Involved **concepts**:

- Conditional beliefs
  - Believe the box is empty
  - But if it's *not* empty, it most likely contains a gift
- Actions and perception
  - Drop box $\rightarrow$ fragile objects breaks
  - Clink $\rightarrow$ presumably something broke

What's in the box? — Thesis objective

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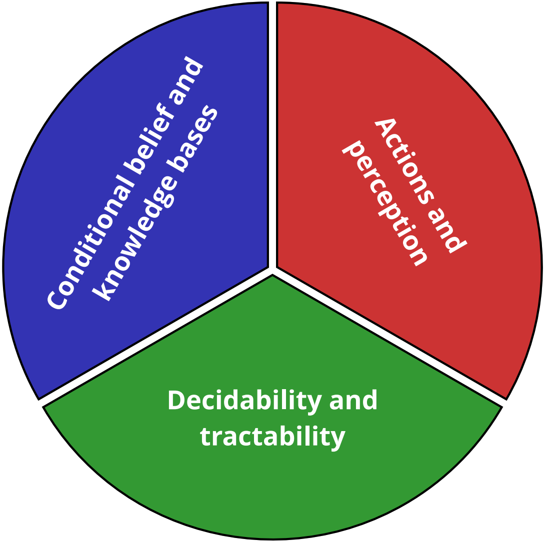
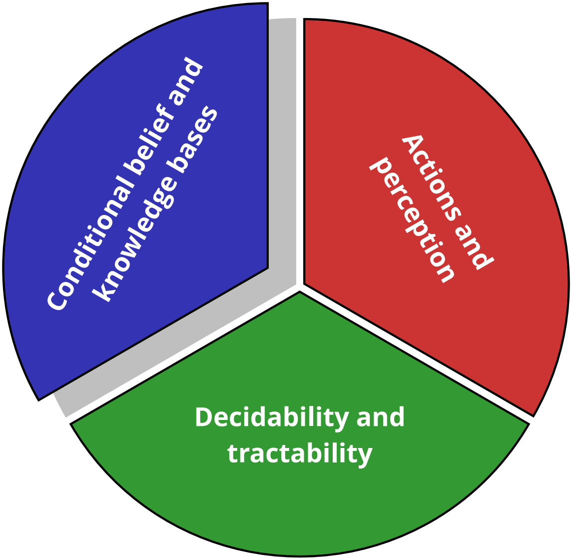
Thesis **objective**:

- Formalize these concepts
- Reason about them effectively

Key **questions**:

- What is a conditional knowledge base?
- How are beliefs affected by actions and perception?
- When is reasoning computationally feasible?


Conditional belief and knowledge bases
Actions and perception
Decidability and tractability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What does "believe that if α, then also β" mean?

Believing a **material implication** is insufficient:

- Semantics: $\neg\alpha \vee \beta$ is believed
- Vacuously true when $\neg\alpha$ is believed
- Often counterintuitive
  - ✗ If the box is *not* empty, there's peace on Earth

Conditional belief is more intuitive:

- Rank possible worlds by plausibility
- Semantics: β holds in the most-plausible α-worlds
- Meaningful even when $\neg\alpha$ is believed

Logic of conditional belief

First-order logic with two modal operators:

- $\mathbf{B}(\alpha \Rightarrow \beta)$ $\hat{=}$ we believe that if α, then β
- $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ $\hat{=}$ *all* we believe is $\{\alpha_i \Rightarrow \beta_i\}$ a.k.a. only-believing
- Here: no nested beliefs

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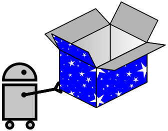
Belief implication

Does $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ entail $\mathbf{B}(\alpha \Rightarrow \beta)$?

- Generalizes Levesque's *logic of only-knowing* to conditional belief
- Next: semantics and properties

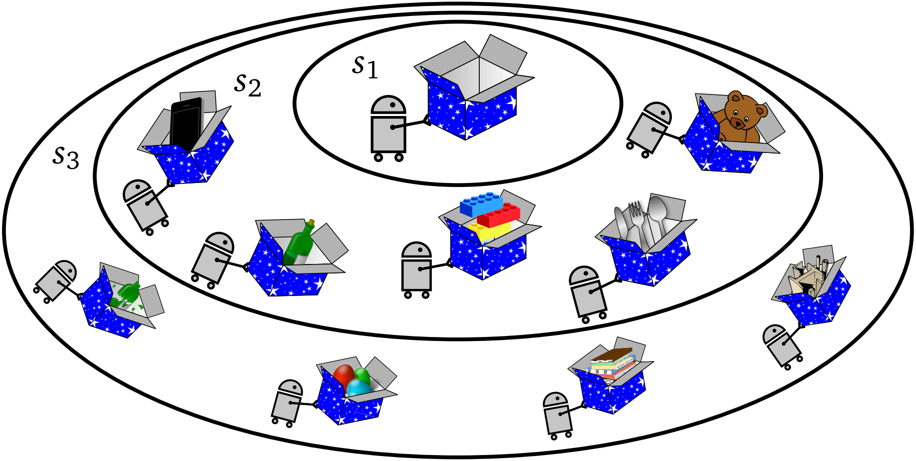
Worlds and systems of spheres

- A **world** is a truth assignment



Worlds and systems of spheres

- A **world** is a truth assignment
- A **system of spheres** $\vec{s}$ ranks possible worlds by plausibility



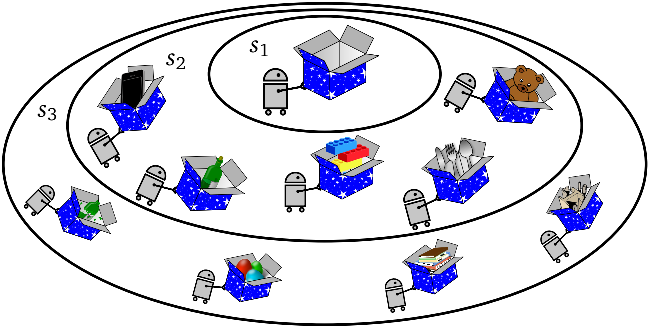
Conditional believing

Semantics

$\vec{s}$ satisfies $\mathbf{B}(\alpha \Rightarrow \beta)$ *iff* the first sphere of $\vec{s}$ consistent with α satisfies $\alpha \supset \beta$

$$\mathbf{B}(\text{True} \Rightarrow \forall x\, \neg \text{InBox}(x))$$

$$\mathbf{B}(\exists y\, \text{InBox}(y) \Rightarrow \exists x\, (\text{InBox}(x) \land \text{Gift}(x)))$$



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Some properties

- Believing $\alpha \;\hat{=}\; \mathbf{B}(\text{True} \Rightarrow \alpha)$
- Knowing $\alpha \;\hat{=}\; \mathbf{B}(\neg\alpha \Rightarrow \text{False})$
- Quantifying-in $\neg\exists x\, \mathbf{B}(\exists y\, \text{InBox}(y) \Rightarrow \text{InBox}(x))$
- Non-monotonic

Conditional only-believing

Semantics

$\vec{s}$ satisfies $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ *iff* $\vec{s}$ is maximal such that $\vec{s}$ satisfies all $\mathbf{B}(\alpha_i \Rightarrow \beta_i)$

- $\vec{s}$ is maximal $\hat{=}$ no worlds can be added to any sphere without removing worlds from some sphere
- $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ $\hat{=}$ *all* we believe is $\{\alpha_i \Rightarrow \beta_i\}$

Unique-model property

A unique $\vec{s}$ satisfies $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$

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- Suppose $\vec{s}$ and $\vec{s}'$ satisfy $\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$
- $C_p := \{\alpha_i \supset \beta_i \mid s_p \text{ is first sphere consistent with } \alpha_i\}$
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- Lemma 1: $s_p = \{w \mid w \text{ satisfies } C_p \cup I_p\}$
- Lemma 2: $C_{p+1} \cup I_{p+1} = I_p$
- Lemma 3: $s_p = s'_p$ implies $I_p = I'_p$

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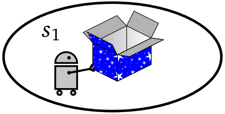

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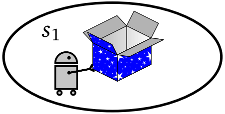

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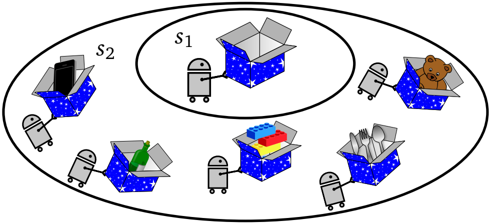

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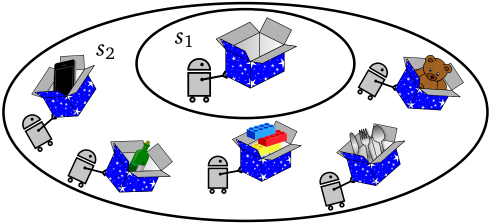

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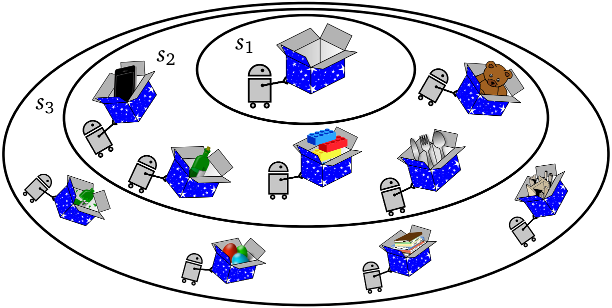

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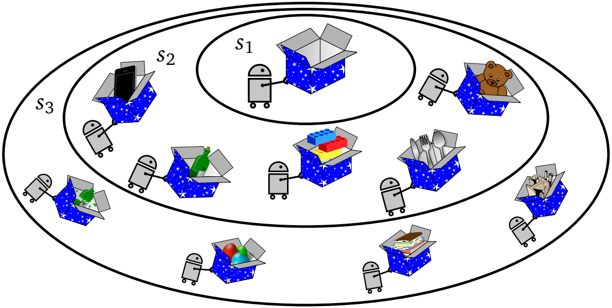
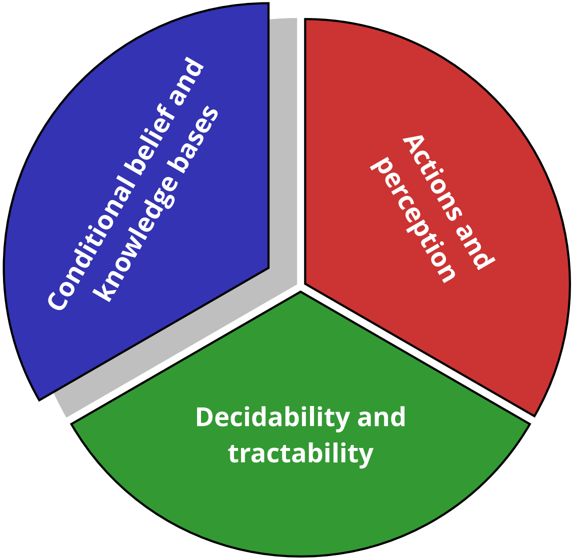


Contribution: conditional belief and knowledge bases

- Conditional belief $\rightarrow$ more and less plausible beliefs
- Only-believing captures idea of conditional KB

- Generalized Levesque's *logic of only-knowing* to conditional belief
  - Our logic subsumes Levesque's
  - Unique-model property of only-believing
  - Levesque's representation theorem extends nicely: decide belief implication with non-modal reasoning
- Related to Pearl's *System Z*
  - System Z is a meta-logical framework
  - Our logic subsumes Pearl's 1-entailment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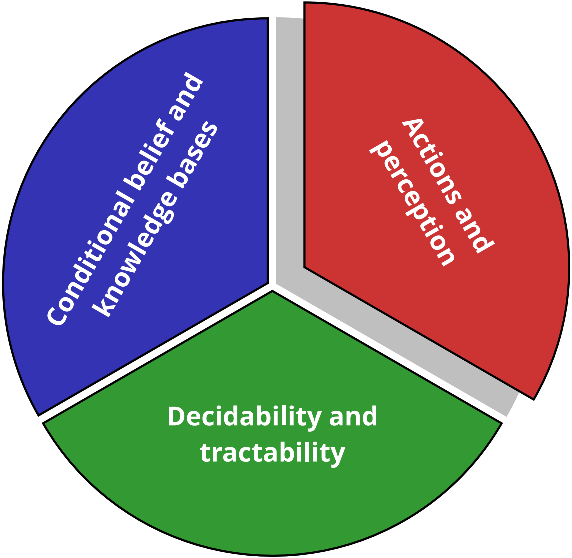
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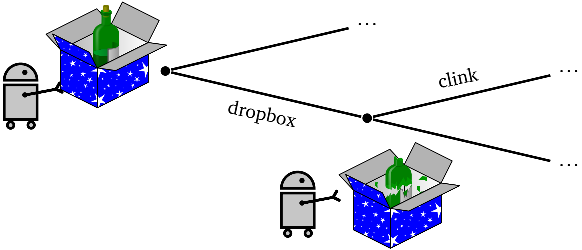
Action effects: physical and/or epistemic

Physical effect:

- Drop box → fragile objects breaks
- Semantics: update every single world



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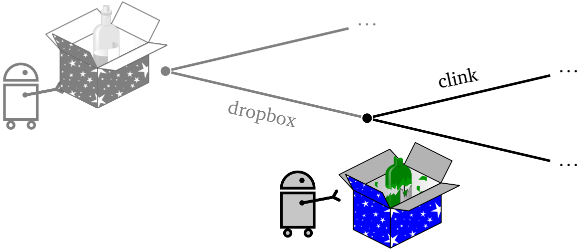
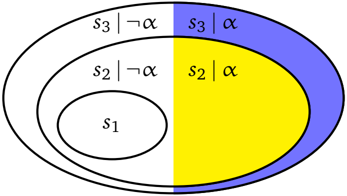


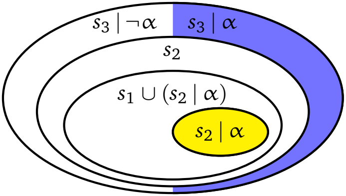
Action effects: physical and/or epistemic

Epistemic effect:

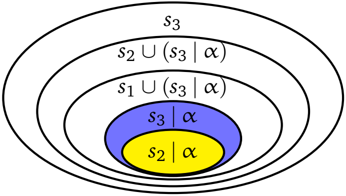
- Clink $\rightarrow$ presumably something broke
- Semantics: revise the system of spheres



Original system of spheres



Weak revision by α



Strong revision by α

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Actions **inform** the agent:

- Action A tells that φ_A is presumably true
- φ_A is incorporated by weak or strong revision
- Contradicting information is no problem

Situation calculus with conditional belief

Two new modal operators:

- $[A]\alpha \;\hat{=}\; \alpha$ holds after action A
- $\Box\alpha \;\hat{=}\; \alpha$ holds always

Action theory $\mathbf{O}(\Sigma_{\text{bel}} \cup \Sigma_{\text{dyn}})$:

- $\Sigma_{\text{bel}} \;\hat{=}\;$ initial beliefs
- $\Sigma_{\text{dyn}} \;\hat{=}\;$ knowledge about dynamics

  $\Box\forall x\,([\text{dropbox}]\text{Broken}(x) \equiv \text{Broken}(x) \vee (\text{Fragile}(x) \wedge \text{InBox}(x)))$

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Belief projection

Does $\mathbf{O}(\Sigma_{\text{bel}} \cup \Sigma_{\text{dyn}})$ entail $[A_1]\ldots[A_k]\mathbf{B}(\alpha \Rightarrow \beta)$?

- Based on Lakemeyer and Levesque's *epistemic situation calculus*
- Solution: reduce belief projection to belief implication

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  - Regression: roll back actions $A_k,\ldots,A_1$ in query
  - Progression: apply effects of $A_1,\ldots,A_k$ to initial beliefs Σ_{bel}

Projection by regression

Correctness of regression

$\mathbf{O}(\Sigma_{\text{bel}} \cup \Sigma_{\text{dyn}})$ entails α *iff* $\mathbf{O}\Sigma_{\text{bel}}$ entails $\mathcal{R}[\alpha]$

$\mathcal{R}[\alpha]$ obtained by repeating until no $[A]$ operator is left:

1. Push $[A]$ operators inwards
2. **Predicates**: axioms Σ_{dyn} relate truth after and before A
   $[\text{dropbox}]\text{Broken}(x) \mapsto \text{Broken}(x) \vee (\text{Fragile}(x) \wedge \text{InBox}(x))$
3. **Beliefs**: theorems relate belief after and before A
   $$\begin{aligned}[A]\mathbf{B}(\alpha \Rightarrow \beta) \mapsto\ & \mathbf{B}(\varphi_A \wedge [A]\alpha \Rightarrow [A]\beta) \wedge \neg\mathbf{B}(\varphi_A \Rightarrow \neg[A]\alpha) \vee \\ & \mathbf{B}(\quad\quad [A]\alpha \Rightarrow [A]\beta) \wedge \quad \mathbf{B}(\varphi_A \Rightarrow \neg[A]\alpha) \vee \\ & \mathbf{B}(\varphi_A \Rightarrow \text{False})\end{aligned}$$

Projection by regression

Correctness of regression

$\mathbf{O}(\Sigma_{\text{bel}} \cup \Sigma_{\text{dyn}})$ entails α *iff* $\mathbf{O}\Sigma_{\text{bel}}$ entails $\mathcal{R}[\alpha]$

$\mathcal{R}[\alpha]$ obtained by repeating until no $[A]$ operator is left:

1. Push $[A]$ operators inwards
2. **Predicates**: axioms Σ_{dyn} relate truth after and before A
   $[\text{dropbox}]\text{Broken}(x) \mapsto \text{Broken}(x) \vee (\text{Fragile}(x) \wedge \text{InBox}(x))$
3. **Beliefs**: theorems relate belief after and before A

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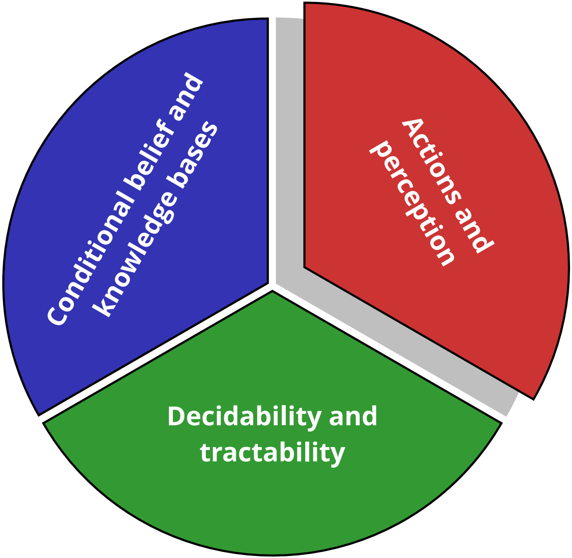
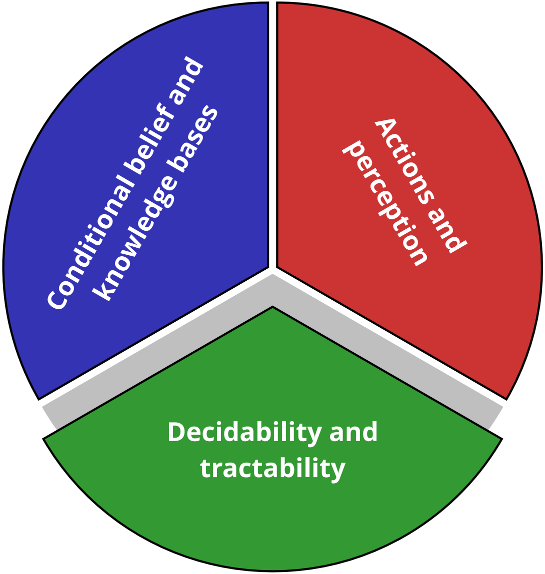
Contribution: actions and perception

- Conditional belief + situation calculus
- Physical and/or epistemic (informing) effect
- Projection by regression and progression

- Based on Lakemeyer and Levesque's epistemic sitcalc
  - We handle contradicting information
  - Extended regression, progression for conditional belief
- Goes beyond Shapiro et al.'s sitcalc with belief change
  - Our logic supports proper revision
  - We address belief projection


Conditional belief and knowledge bases
Actions and perception
Decidability and tractability


Conditional belief and knowledge bases
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Why decidable reasoning?

Our logic is **too powerful**:

- Omniscient
- Undecidable (first-order) / intractable (propositional)

Possible **approaches**:

- Restrict expressivity (classical approach)
- Restrict inferences

Limited reasoning:

- Set of worlds $\rightarrow$ setup: set of ground clauses closed under subsumption and unit propagation
- Add literals $\rightarrow$ new inferences

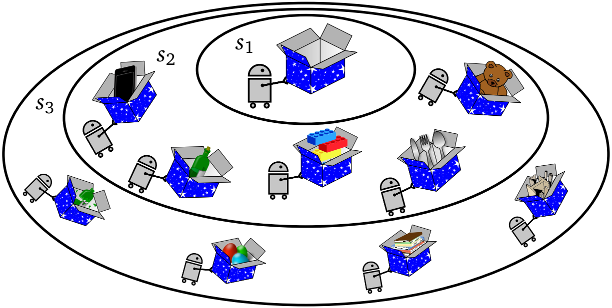
Add literals $\hat{=}$ Effort

subsumption: $A \vee B$ subsumes $A \vee B \vee C$

unit propagation: $\neg A$ and $A \vee B \vee C$ yield $B \vee C$

Example of limited conditional belief

$$\mathbf{O}\{\mathrm{True} \Rightarrow \forall x\, \neg \mathrm{InBox}(x),$$
$$\exists y\, \mathrm{InBox}(y) \Rightarrow \forall x\, (\mathrm{InBox}(x) \supset \mathrm{Gift}(x)),$$
$$\exists y\, \mathrm{InBox}(y) \Rightarrow \forall x\, (\mathrm{InBox}(x) \supset \neg \mathrm{Broken}(x))\}$$



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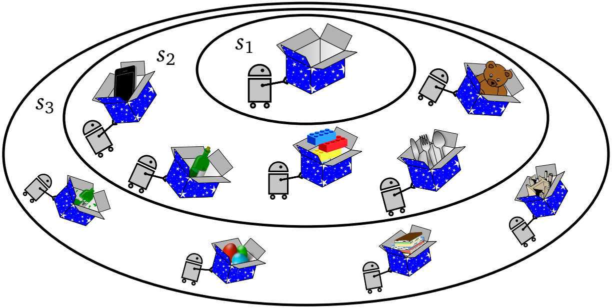
$$s_1 = \{\neg \text{InBox}(n) \mid n \in N\}$$
$$s_2 = \{\neg \text{InBox}(n) \vee \text{Gift}(n),\ \neg \text{InBox}(n) \vee \neg \text{Broken}(n) \mid n \in N\}$$
$$s_3 = \{\}$$

set of individuals



Example of limited conditional belief

$$s_1 = \{\neg\mathrm{InBox}(n) \mid n \in N\}$$
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Does $\vec{s}$ satisfy $\mathbf{B}(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \land \neg\mathrm{Broken}(n))$?

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- s_1 is not consistent with $\mathrm{InBox}(n)$
  - s_1 contains $\neg \mathrm{InBox}(n)$

Example of limited conditional belief

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Does $\vec{s}$ satisfy $\mathbf{B}(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \wedge \neg \mathrm{Broken}(n))$?

- s_2 is consistent with $\mathrm{InBox}(n)$
  - $s_2 \cup \{\mathrm{InBox}(n)\}$ is consistent
  - $s_2 \cup \{\mathrm{InBox}(n)\}$ contains $\mathrm{InBox}(n)$

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level $\hat{=}$ added literals

Does $\vec{s}$ satisfy $\mathbf{B}_1(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \wedge \neg \mathrm{Broken}(n))$? ✓

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level $\hat{=}$ added literals

Does $\vec{s}$ satisfy $\mathbf{B}_0(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \wedge \neg \mathrm{Broken}(n))$? ✗

Does $\vec{s}$ satisfy $\mathbf{B}_1(\mathrm{InBox}(n) \Rightarrow \mathrm{Gift}(n) \wedge \neg \mathrm{Broken}(n))$? ✓

- s_2 is consistent with $\mathrm{InBox}(n)$
  - $s_2 \cup \{\mathrm{InBox}(n)\}$ is consistent
  - $s_2 \cup \{\mathrm{InBox}(n)\}$ contains $\mathrm{InBox}(n)$
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Logic of limited conditional belief

Belief operators with effort $k \in \{0, 1, 2, \ldots\}$

- $\mathbf{B}_k(\alpha \Rightarrow \beta)$ $\hat{=}$ belief at level k
- $\mathbf{O}_k\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ $\hat{=}$ only-belief at level k
  α_i only mentions $\wedge$, $\exists$, literals
  β_i only mentions $\vee$, $\forall$, literals
  } prenex-NNF of $\alpha_i \supset \beta_i$ is a clause

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Limited belief implication

Does $\mathbf{O}_k\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ entail $\mathbf{B}_{k'}(\alpha \Rightarrow \beta)$?

- Based on Liu, Lakemeyer, Levesque's *limited knowledge*
- Semantics for $\mathbf{B}_k$ and $\mathbf{O}_k$ mostly as for $\mathbf{B}$ and $\mathbf{O}$ except:
  - Sets of possible worlds $\mapsto$ setups
  - Sound but incomplete consistency and satisfaction
- Next: soundness and decidability

Limited belief implication: soundness and decidability

Soundness

If $\mathbf{O}_k\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ entails $\mathbf{B}_{k'}(\alpha \Rightarrow \beta)$, then
$\mathbf{O}\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ entails $\mathbf{B}(\alpha \Rightarrow \beta)$

Limited belief implication: soundness and decidability

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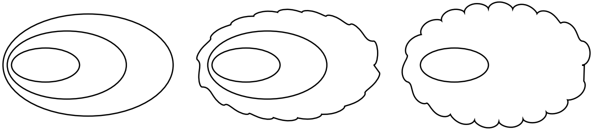

Why?

- $\mathbf{O}_k$'s first spheres are faithful to $\mathbf{O}$'s spheres
- $\mathbf{O}_k$'s last sphere believes less than $\mathbf{O}$'s sphere
- $\mathbf{B}_k$ doesn't select a too-narrow sphere

Limited belief implication: soundness and decidability

Complexity

Whether $\mathbf{O}_k\{\alpha_1 \Rightarrow \beta_1, \ldots, \alpha_m \Rightarrow \beta_m\}$ entails $\mathbf{B}_{k'}(\alpha \Rightarrow \beta)$ is

- First-order case: decidable
- Propositional case: tractable for fixed effort k, k'

Limited belief implication: soundness and decidability

Complexity

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- First-order case: decidable
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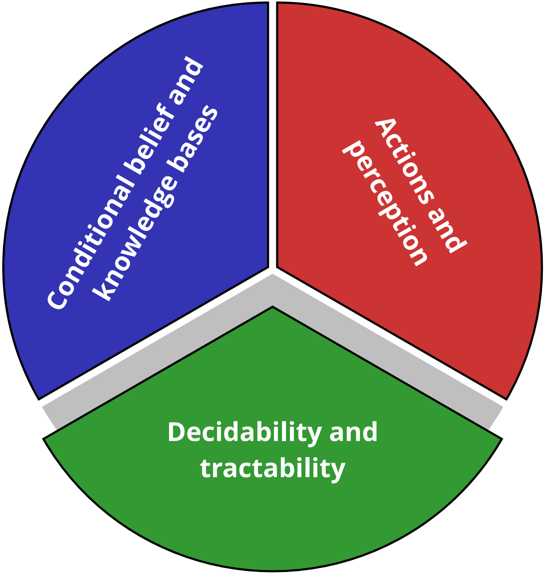
Why?

- Unique system of spheres since $\alpha_i \supset \beta_i$ are clauses
- Only finitely many individuals can be distinguished by formulas
- Only finitely many literals are relevant for adding

Contribution: limited conditional belief

- Effort bounds possible inferences
- Limited belief implication is decidable, sound
- Sacrificed completeness, preserved expressivity

- Based on Liu, Lakemeyer, Levesque's limited knowledge
  - Added sound consistency test
  - Approximative system of spheres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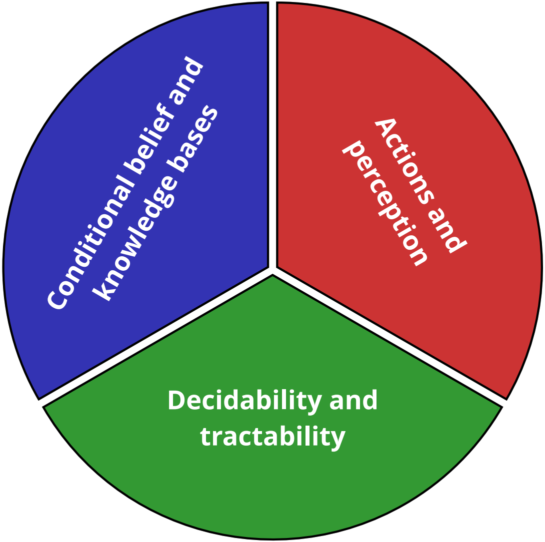
Conditional belief and knowledge bases
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Conditional Beliefs in Action

- What is a conditional knowledge base?
- How are beliefs affected by actions and perception?
- When is reasoning computationally feasible?

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- When is reasoning computationally feasible?
  - Sound, decidable, sometimes tractable belief implications
  - Sacrificed completeness, preserved expressivity

Summary and future work

Conditional Beliefs in Action

- What is a conditional knowledge base?
  - Logic of conditional only-believing
  - Generalizes Levesque's logic, subsumes Pearl's 1-entailment
  - Next: Probabilities?
- How are beliefs affected by actions and perception?
  - Sitcalc-style actions with belief revision
  - Projection by regression and progression
  - Next: More revision operators?
- When is reasoning computationally feasible?
  - Sound, decidable, sometimes tractable belief implications
  - Sacrificed completeness, preserved expressivity
  - Next: Tractable revision by limited reasoning?