Limbo

Reasoning in the **Situation Calculus** with **Limited Belief**

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Why Limited Belief?

Task: Robot has a KB and a query:

Does the KB *logically entail* the query **?**

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Which logic?

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Which logic?

Classical logic:

- Unrealistic: omniscient agent
- Undecidable (first-order) / intractable (propositional)

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Task: Robot has a KB and a query:

Does the KB *logically entail* the query **?**

(*logically entail*: Which logic?)

Limited belief: [Lakemeyer & Levesque, KR-2016]

- Belief level 0: explicit beliefs KB **+** unit propagation **+** subsumption
- Belief level $k+1$: implicit beliefs belief level k **+** one case split

Hypothesis: good results at *small* belief level

Limbo: A Reasoning System [Schwering, IJCAI-2017]

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  - Logical reasoning in a very expressive language
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  - Introspective belief
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  - Actions ⟵ this paper

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  - Sudoku: level 0 $\approx$ easy; 1 $\approx$ medium; 2 $\approx$ hard; 4 $\approx$ extreme

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  - Sudoku: level 0 $\approx$ easy; 1 $\approx$ medium; 2 $\approx$ hard; 4 $\approx$ extreme
  - Minesweeper: level 0 is bad; 1 is good; 2 slightly better; 3 same

Situation Calculus [Reiter 2001; Lakemeyer & Levesque, AIJ-2011]

- Logical language for actions, sensing, knowledge
- Values of functions / predicates change in situations
- Situation is a sequence of actions
- Basic action theory:
  - successor-state axioms capture effects of actions
  - sensing axiom captures how knowledge is produced
  - initial theory describes the initial state
- Projection: does a BAT entail a query after certain actions?

The Language

FOL with equality **+** functions **+** sorts **+**

- Knowledge: $\mathbf{K}_0\,\alpha \quad \mathbf{K}_1\,\alpha \quad \mathbf{K}_2\,\alpha \quad \ldots$
- Possibility: $\mathbf{M}_0\,\alpha \quad \mathbf{M}_1\,\alpha \quad \mathbf{M}_2\,\alpha \quad \ldots$
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Examples:

- $[\mathrm{birth}(\mathrm{Mia}, \mathrm{Sally})]\big(\mathrm{motherOf}(\mathrm{Sally}) = \mathrm{Mia} \land \mathrm{fatherOf}(\mathrm{Sally}) = \mathrm{Frank}\big)$

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- $[\mathrm{birth}(\mathrm{Mia},\mathrm{Sally})][\mathrm{test}(\mathrm{Mia},\mathrm{Fred})]\mathbf{K}_1\,\mathrm{fatherOf}(\mathrm{Sally}) = \mathrm{Frank}$

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standard names represent distinct individuals

The Semantics

Model: set of possible worlds

Belief: true in all possible worlds

The Semantics

disjunction of literals $[A_1] \ldots [A_j](\neg)t_1 = t_2$

Model: set of **clauses** closed under unit propagation

- Belief level 0: subsumption
- Belief level $k+1$: case split **+** level k

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Example:

If we know that (a) $\text{spouseOf}(\text{Mia}) = \text{Frank} \lor \text{spouseOf}(\text{Mia}) = \text{Fred}$

and (b) $\text{motherOf}(\text{Sally}) = \text{Mia}$

and (c) $\text{motherOf}(y) \neq z \lor \text{spouseOf}(z) \neq x \lor \text{fatherOf}(y) = x$

then $\mathbf{K}_{\mathbf{0}} \big(\text{fatherOf}(\text{Sally}) = \text{Frank} \lor \text{fatherOf}(\text{Sally}) = \text{Fred}\big)$ **?**

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then $\mathbf{K_0}\big(\text{fatherOf}(\text{Sally}) = \text{Frank} \lor \text{fatherOf}(\text{Sally}) = \text{Fred}\big)$ **?**

No! No known clause subsumes $\text{fatherOf}(\text{Sally}) = \text{Frank} \lor \text{fatherOf}(\text{Sally}) = \text{Fred}$.

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and (c) $\text{motherOf}(y) \neq z \lor \text{spouseOf}(z) \neq x \lor \text{fatherOf}(y) = x$

then $\mathbf{K}_1 \left(\text{fatherOf}(\text{Sally}) = \text{Frank} \lor \text{fatherOf}(\text{Sally}) = \text{Fred}\right)$ **?**

Yes! Branch on $\text{spouseOf}(\text{Mia})$:

- $\{$(a), (b), (c), $\text{spouseOf}(\text{Mia}) = \text{Frank}\} \ni \text{fatherOf}(\text{Sally}) = \text{Frank}$ by UP with (b), (c)
- $\{$(a), (b), (c), $\text{spouseOf}(\text{Mia}) = \text{Fred}\} \ni \text{fatherOf}(\text{Sally}) = \text{Fred}$ by UP with (b), (c)
- $\{$(a), (b), (c), $\text{spouseOf}(\text{Mia}) = n\} \ni \bot$ by UP with (a) for $n \neq \text{Frank}, \text{Fred}$

Proper$^+$ Knowledge Bases

Let KB be a conjunction of clauses

$$\forall \vec{x} (\ell_1 \vee \ldots \vee \ell_j)$$

$$\Box \forall \vec{x} (\ell_1 \vee \ldots \vee \ell_j)$$

where ℓ_i is of the form $[A_1] \ldots [A_l] (\neg) t_1 = t_2$

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KB entails query at a belief level $\Longrightarrow$ KB entails query classically

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Theorem: Eventual Completeness

If KB and query contain no $\exists, \forall, \Box$:

KB entails query classically $\Longrightarrow$ KB entails query at a belief level

Basic Action Theories

Contains successor-state axioms

$$\square\big([a]\,f(x_1,\ldots,x_j) = y \equiv \phi_f\big)$$

And a sensed-fluent axiom

$$\square\big(\mathrm{sf}(a) = y \equiv \psi\big)$$

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Example:

$$\Box\big([a]\mathrm{motherOf}(x) = y \equiv a = \mathrm{birth}(y,x) \vee a \neq \mathrm{birth}(y,x) \wedge \mathrm{motherOf}(x) = y\big)$$

$$\Box\big([a]\mathrm{fatherOf}(x) = y \equiv \exists\hat{y}\,(a = \mathrm{birth}(\hat{y},x) \wedge \mathrm{spouseOf}(\hat{y}) = y) \vee \forall\hat{y}\,(a \neq \mathrm{birth}(\hat{y},x) \wedge \mathrm{fatherOf}(x) = y)\big)$$

$$\Box\big(\mathrm{sf}(a) = y \equiv \exists x\,\exists\hat{y}\,(a = \mathrm{test}(\hat{y},x) \wedge \mathrm{fatherOf}(x) = \hat{y}) \wedge y = \mathrm{Yes} \vee \forall x\,\forall\hat{y}\,(a \neq \mathrm{test}(\hat{y},x) \vee \mathrm{fatherOf}(x) \neq \hat{y}) \wedge y = \mathrm{No}\big)$$

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If KB is a BAT translated into proper^{+} form, and query is $\Box$-free:

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Theorem: Tractability

If KB is a BAT translated into proper^{+} form, and query is $\square$-free, and both contain no $\exists, \forall$:

KB entails query at a belief level is tractable

Regression

Eliminates actions:

1. Push actions $[A]$ inwards
2. **Functions**: axioms relate truth after and before A
   $[A]\, f(t_1, \ldots, t_j) = t_{j+1} \;\mapsto\; \phi_f{}^{a\; x_1\; \ldots\; x_j\; y}_{A\; t_1\; \ldots\; t_j\; t_{j+1}}$
3. **Knowledge**: theorems relate knowledge after and before A
   $[A]\mathbf{K}_k\,\alpha \;\mapsto\; \forall x\,\big(\mathrm{sf}(A) = x \supset \mathbf{K}_k\,(\mathrm{sf}(A) = x \supset [A]\alpha)\big)$
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Summary

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Limbo implements limited belief and actions

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Practical challenges:

- Keep up with the theory
- Improve performance
- Find applications

Appendix

Experiments: Sudoku Minesweeper

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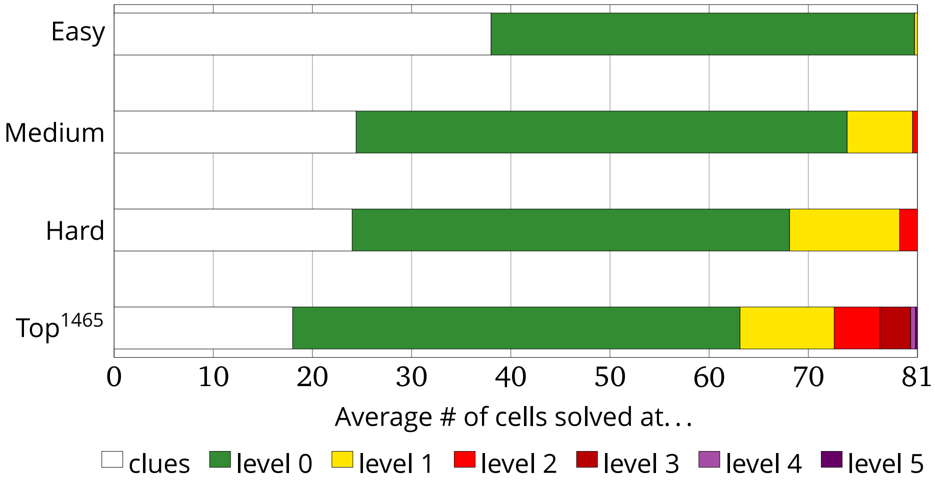

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