<u>First-order multi-agent epistemic logic</u>:

$\mathbf{K}_A\alpha$ = A knows α
$\mathbf{O}_A\alpha$ = A knows α and nothing else

<u>Example</u>:

- A knows $\boxed{A}$
- A knows that → B knows $\boxed{B}$

A knows that she does not know $\boxed{B}$ and that B does know $\boxed{B}$.

<u>Formalisation</u>:

$$\mathbf{O}_A\big(\boxed{A} = 7 \land \forall x\big(\boxed{B} = x \rightarrow \mathbf{O}_B\boxed{B} = x\big)\big)$$

entails

$$\mathbf{K}_A\exists z\big(\neg\mathbf{K}_A\boxed{B} = z \land \mathbf{K}_B\boxed{B} = z\big)$$

<u>Reduction using classical FOL validity oracle</u>:

- Replace each $\mathbf{O}_A\alpha(\vec{x})$ with a fresh atom $P_\alpha(\vec{x})$
- Replace each $\mathbf{K}_A\gamma(\vec{z})$ with a disjunction of

$$\exists\vec{x}\big(P_\alpha(\vec{x}) \land \text{Valid}[\alpha(\vec{x}) \rightarrow \gamma(\vec{z})]\big)$$

over all $\mathbf{O}_A\alpha(\vec{x})$

- Add axioms for $P_\alpha(\vec{x}), P_\beta(\vec{y})$

$$P_\alpha(\vec{x}) \rightarrow \big(P_\beta(\vec{y}) \leftrightarrow \text{Valid}[\alpha(\vec{x}) \rightarrow \beta(\vec{y})]\big)$$

to mimic properties of $\mathbf{O}_A$

Valid$[\phi(\vec{x})]$ represents the $\vec{x}$ for which ϕ is valid

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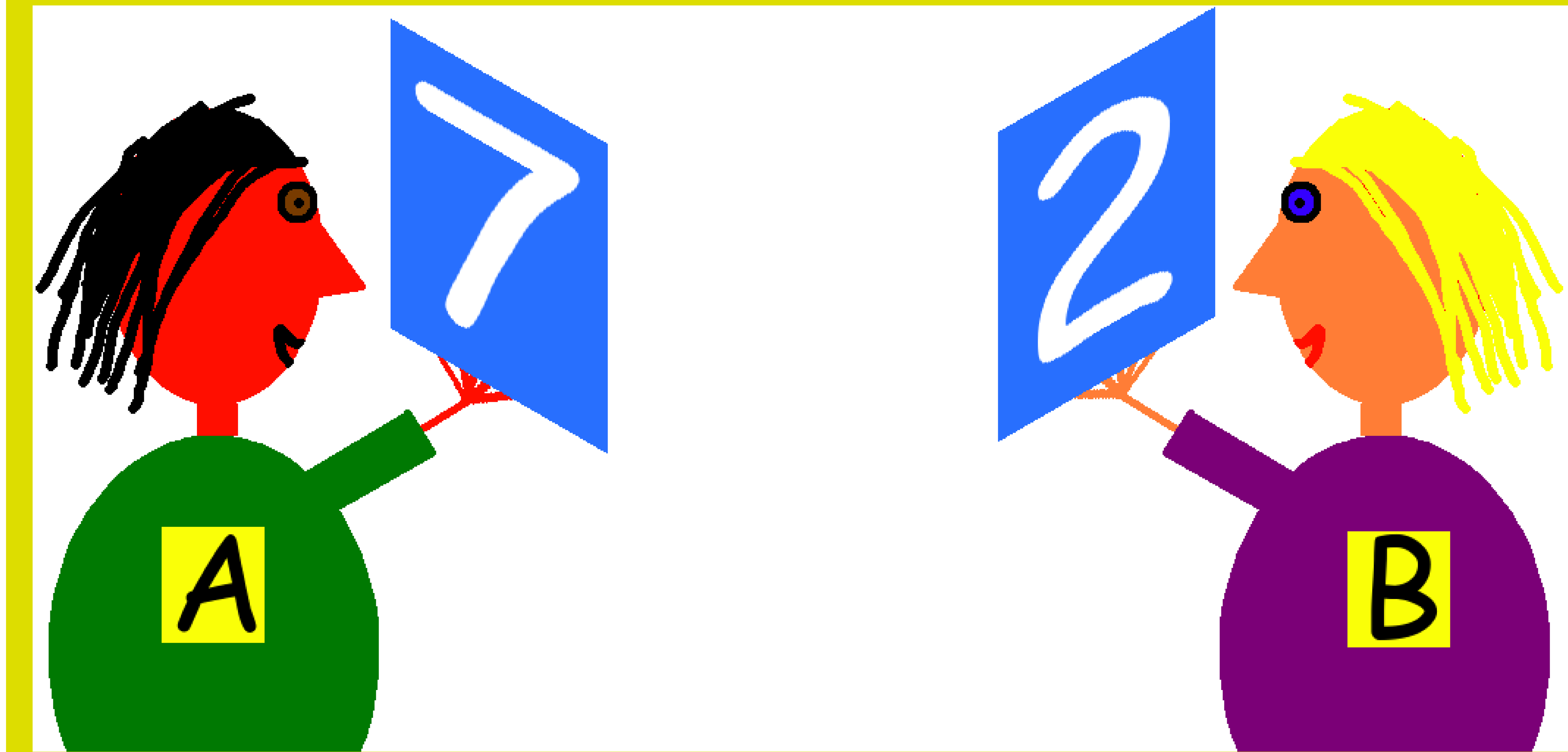

Reasoning in <u>multi-agent epistemic KBs</u> reduces to <u>classical reasoning</u>

$\mathbf{O}_A\alpha$ is a <u>multi-agent knowledge base</u>
$\iff$ every model of α satisfies some $\mathbf{O}_B\beta$

✗ $\mathbf{O}_A\big(p \rightarrow \mathbf{O}_B q\big)$
✓ $\mathbf{O}_A\big((p \rightarrow \mathbf{O}_B q) \land (\neg p \rightarrow \mathbf{O}_B r)\big)$
✓ $\mathbf{O}_A\forall x\big(p(x) \rightarrow \mathbf{O}_B q(x)\big)$

Allows for incomplete knowledge about other agent's knowldege!

For objective ϕ, ψ:
$\mathbf{O}_A\phi$ entails $\mathbf{K}_A\psi$ $\iff$ $\phi \rightarrow \psi$ is valid

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$\mathbf{O}_A\alpha$ implies $\mathbf{O}_A\beta$ $\iff$ α, β equivalent

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<u>Reduction using classical FOL validity oracle</u>:

1. $\mathbf{O}_A\big(\boxed{A} = 7 \land \forall x\big(\boxed{B} = x \rightarrow \mathbf{O}_B\boxed{B} = x\big)\big)$

   $\mathbf{K}_A\exists z\big(\neg\mathbf{K}_A\boxed{B} = z \land \mathbf{K}_B\boxed{B} = z\big)$
2. $\mathbf{O}_A\big(\boxed{A} = 7 \land \forall x\big(\boxed{B} = x \rightarrow P(x)\big) \land \Omega\big)$

   $\mathbf{K}_A\exists z\big(\neg\mathbf{K}_A\boxed{B} = z \land \exists x(P(x) \land x = z)\big)$

When is $\boxed{B} = x \rightarrow \boxed{B} = z$ valid?
Use FOL validity oracle!

$$\Omega = \forall x\forall y(P(x) \rightarrow (P(y) \leftrightarrow x = y))$$

<u>Oracle calls</u>: [Levesque '84]

- if ϕ has no free variables:
  Valid$[\phi]$ = TRUE if ϕ valid
  Valid$[\phi]$ = FALSE otherwise
- if ϕ has free variable x, names $n_1, \ldots, n_k$:
  Valid$[\phi]$ = (Valid$[\phi^x_{n_1}] \land x = n_1) \lor \ldots \lor$
  (Valid$[\phi^x_{n_k}] \land x = n_k) \lor$
  (Valid$[\phi^x_{n'}]^{n'}_x \land x \neq n_1 \land \ldots \land x \neq n_k$)