Projection in the Epistemic Situation Calculus with Belief Conditionals

Christoph Schwering and Gerhard Lakemeyer

RWTH Aachen University, Germany

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Belief Revision in Dynamic Environments

Suppose we want to have dinner at a restaurant:

- We don't know that the restaurant is Italian
- We believe:
  1. usually, the specialty is burger
  2. but in Italian rest.s, it's pizza or pasta
- We can take action:
  1. order the specialty
  2. ask if restaurant is Italian

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Belief projection: After actions $n_1, \ldots, n_k$, do we believe α?

E.g.: After we order the specialty and then find out the restaurant is Italian, do we believe that we will get a dish but don't know which?

Belief Revision in the Situation Calculus

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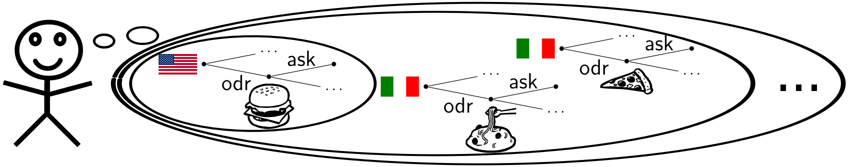
E.g.: Truth in a model (in a variant of Shapiro et al. [AIJ-2011]):

$$f, w \models [\text{odr}][\text{ask}]\mathbf{B}(\exists x. D(x) \wedge \neg \mathbf{B} D(x))$$



Belief Revision in the Situation Calculus

Suppose we want to have dinner at a restaurant:

- We don't know that the restaurant is Italian
- We believe:
  1. usually, the specialty is burger $\textsf{TRUE} \Rightarrow S(x) \equiv x = \text{burger}$
  2. but in Italian rest.s, it's pizza or pasta $I \Rightarrow S(\text{pizza}) \vee S(\text{pasta})$

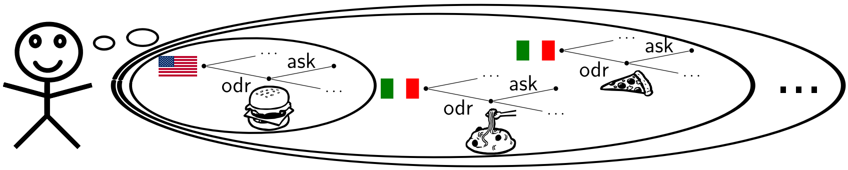
  $\Big\} = \Gamma$
- We can take action:
  1. order the specialty $\Box[a]D(x) \equiv a = \text{odr} \wedge S(x) \vee D(x)$
  2. ask if restaurant is Italian $\Box SF(a) \equiv a = \text{ask} \supset I$

  $\Big\} = \Omega$

Belief projection: After actions $n_1, \ldots, n_k$, do we believe α?

E.g.: Entailments of action theory:

$$\Omega \wedge I \wedge \mathbf{O}(\Omega, \Gamma) \models [\text{odr}][\text{ask}]\mathbf{B}(\exists x. D(x) \wedge \neg \mathbf{B} D(x))$$



Belief Revision in the Situation Calculus

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Solution: 1. elimination of actions using regression

2. elimination of beliefs by reduction to first-order reasoning

Elimination of Actions: The Objective Case

Regression (due to Reiter):

- Push actions inwards
- Replace $[r]F(\vec{t})$ and $SF(r)$ with the RHS from the action theory
- Yields a formula without actions

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$$
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Elimination of Actions: The Subjective Case

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How to push actions inwards of $\mathbf{B}$ modalities?

Theorem:

$$\begin{aligned}\models \Box\, [a]\mathbf{B}(\phi \Rightarrow \psi) \equiv \neg SF(a) \wedge \mathbf{B}(\neg SF(a) \wedge [a]\phi \Rightarrow [a]\psi) \vee \\ SF(a) \wedge \mathbf{B}(\ \ SF(a) \wedge [a]\phi \Rightarrow [a]\psi)\end{aligned}$$

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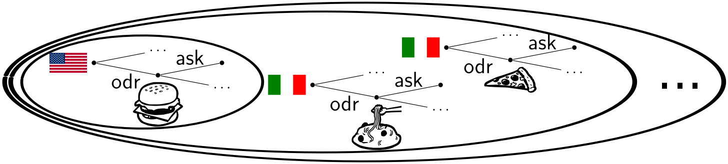
Elimination of Beliefs: The Idea

- $\mathbf{B}(\phi \Rightarrow \psi)$ iff most plausible ϕ-worlds satisfy ψ



- Every sphere can be represented by an objective sentence γ_i
- $\mathbf{B}(\phi \Rightarrow \psi)$ iff first ϕ-consistent γ_i entails $\phi \supset \psi$

$$\begin{aligned}\gamma_0 &= \neg I \wedge (S(x) \equiv x = \text{burger})\\ \gamma_1 &= I \supset (S(\text{pasta}) \vee S(\text{pizza}))\\ \gamma_2 &= \text{TRUE}\end{aligned}$$

- Free variables: enumerate believed instances (due to Levesque)

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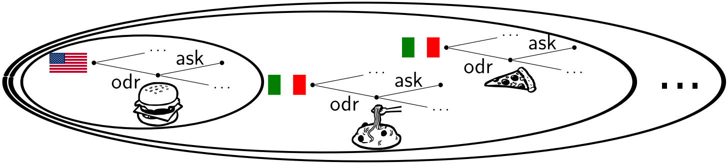

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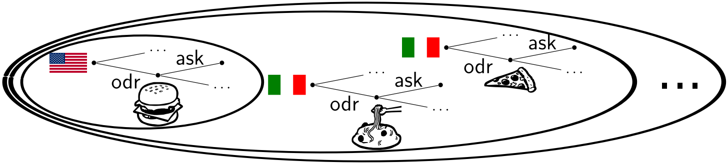

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$$\rightarrow \quad I \wedge \text{TRUE}$$

Conclusion

Solved **belief projection** in the Situation Calculus:

1. Elimination of actions: formula about initial beliefs
2. Elimination of beliefs: series of first-order entailments

Working implementation based on Lakemeyer and Levesque [KR-2014]

Future work:

- Progression of beliefs
- Regression for Spohn-style logics e.g., Delgrande and Levesque [KR-2012]